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Reduction in Adhesive Shear Strains at the Ends of Bonded Reinforcements

T. Tran-Cong and M. Heller

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ABSTRACT

A mathematical model is presented which defines the adhesive shear strain distribution for an adherend with bonded multilayer reinforcements which are stepped at their ends. In this one-dimensional formulation each step is allowed to be of different thickness and modulus, and of variable step length. A procedure is then given to improve the design of such reinforcements through minimising the peak adhesive shear strain which typically occurs near their stepped ends. It is shown that to achieve a 20% reduction in peak adhesive shear strain for a typical stepped patch consisting of unidirectional laminae, the first step adjacent to the patch end needs to be much longer than the remaining steps. For the case where cross-ply laminae are used in conjunction with unidirectional laminae, the maximum shear strain in the adhesive layer can be reduced by about 60%. The results also indicate that reduced peak adhesive shear strains lead to a smoother transition of load from the plate to the patch. This suggests that a patch design which minimises peak adhesive shear strains will also reduce the undesirable stress concentration in the repaired structure, outside the patched region.

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Executive Summary

Bonded composite patches have been widely applied to RAAF aircraft as reinforcements or repairs for cracked structures. These reinforcements function primarily by transferring load from the cracked component to the patch by shear deformation of the adhesive, thereby reducing the magnitude of strains in the repaired component and resulting in increased fatigue lives. For a typical bonded repair configuration, a large peak in the adhesive shear strain occurs at the end of the patch. Currently, uniform stepping of the multi-layer patch is used for bonded repairs undertaken on RAAF aircraft. This approach is recommended in the relevant RAAF Engineering Standard.

In this report, a mathematical model is presented which defines the adhesive shear strain distribution for an adherend with bonded multilayer reinforcements which are stepped at their ends. In this model each step is allowed to be of different thickness and modulus, and of variable step length. A procedure is then given to improve the design of such reinforcements through minimising the peak adhesive shear strain which typically occurs near their stepped ends. It is shown that to achieve a 20% reduction in peak adhesive shear strain for a typical stepped patch consisting of unidirectional laminae, the first step adjacent to the patch end needs to be much longer than the remaining steps. For the case where cross-ply laminae are used in conjunction with unidirectional laminae, the maximum shear strain in the adhesive layer can be reduced by about 60%. The results also indicate that reduced peak adhesive shear strains lead to a smoother transition of load from the plate to the patch. This suggests that that a patch design which minimises peak adhesive shear strains will also reduce the undesirable stress concentration in the repaired structure, outside the patched region.

It is expected that in the future the methodology presented in this report can be incorporated into the RAAF Engineering standard to improve the performance of adhesively bonded composite repairs for RAAF aircraft.

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Contents

Notation

A	Inverse of the characteristic length of the shear stress function.
C_1, C_2	Constants in equation defining shear stresses for a given step k
γ	Shear strain in the adhesive layer.
ε	Direct strain in the x-direction.
δ	Displacement in the x-direction.
E	Young's modulus of adherend.
G	Shear modulus of the adhesive layer.
η	Thickness of the adhesive layer.
L	Total length of the patch.
P	Total tensile force at the end of the plate.
τ	Shear stress in the adhesive layer.
t	Thickness of the current patch step.
T_o	Tensile force in each patch at a given position x.
x,y	Co-ordinates in Cartesian axis system.

Superscripts

k	step number

Subscripts

o	outer adherend (patch)
i	inner adherend (plate)

List of abbreviations

AMRL	Aeronautical and Maritime Research Laboratories
RAAF	Royal Australian Air Force

1. Introduction

Bonded patches, typically consisting of fibre-composite unidirectional laminae, have been successfully used to reduce the level of strains in repaired components such as cracked aircraft structures, as discussed by A. A. Baker and R. Jones [1]. These reinforcements function primarily by transferring load from the cracked component to the patch by shear deformation of the adhesive, thereby reducing the magnitude of strains in the repaired component and resulting in increased fatigue lives. However, there are a number of outstanding issues concerning bonded repairs which need to be addressed. One such issue involves the large peak in the adhesive shear strain which typically occurs near the end of the patch, which can cause failure of the adhesive system and compromise the performance of the repair. To reduce the severity of this peak, uniform stepping of multilayer patches is currently being employed for bonded repairs undertaken on RAAF aircraft. This approach is recommended in the relevant RAAF Engineering Standard, with the typical step length being 3 mm per lamina ply. However, having uniform step lengths is not optimal (in terms of minimising peak adhesive shear strain) and it would seem desirable to investigate whether alternate approaches can be derived.

One possible approach for reducing the peak adhesive strains is to increase the length of the uniform steps of the composite patch, where each step consists of one unidirectional lamina. This method has been considered by Chalkley [3], who found that to achieve a significant benefit as compared to a standard patch configuration, the length of the stepped region needs to be significantly increased. Hence this would result in an increase in the overall length of the patch, which may not be desirable in some applications. In other relevant work, Ojalvo [4] suggested a continuous non-linear tapering of the patch thickness. Implementing this approach can result in the shear strain being maintained at a constant reduced level throughout the length of the patch, which is equivalent to the optimum solution for minimising the peak adhesive shear strain. This approach is particularly appropriate if the patch thickness can be varied continuously, such as in the case of a metallic doubler. However this continuous variation is virtually impossible to achieve if the patch consists of multiple laminae. Rees *et. al.* [5] undertook a finite element analysis of a practical repair application for a multilayer composite patch, where the step length was allowed to be non-uniform, and this was shown to be beneficial. However no analytical formulation or attempt to optimise the stepping scheme was presented.

In this current study, a one dimensional theory is used to formulate the solution to the problem of minimising the peak shear strain in the adhesive for a plate with bonded stepped patches. Here each step is allowed to be of different height and modulus, and of non-uniform step length. The theoretical formulation along with the numerical solution method is presented in Section 2. In Section 3 the results of illustrative numerical analyses are given including comments on the method for minimising the peak adhesive shear strains. Here the approach taken is to limit the various peak values in the adhesive shear strain distribution to one common level. Section 4 discusses the results and their implications and cites recommendations concerning various stepping options.

2. Analysis for plate with symmetric stepped patches

The general configuration of the problem under study is shown in Figure 1. A thin plate of uniform thickness t_i is subjected to a uniaxial remote load P (per unit width in the z direction). Each side of this plate is reinforced with an identical adhesively bonded patch, thereby symmetry is retained with respect to the plate mid-plane $y = 0$, where the origin of the axis system is at the left hand end of the patch The adhesive thickness is uniform and is denoted η. Each patch has a maximum length of L, and is stepped at each of its ends identically, with symmetry being retained with respect to the line $x = L/2$. Its maximum thickness is denoted t_o The geometry and notation for the stepping arrangement is shown in Figure 2. There are n steps at the end of each patch which are allowed to be of different thickness, modulus and of non-uniform length. For an arbitrary step k, the thickness is denoted t_o^k, the modulus is E_o^k, and the position of the beginning of the step is denoted $x_{(k-1)}$. It should be noted here that the step height is defined as the difference in total step thickness from one step to another, ie $t_o^k - t_o^{k-1}$ is the height for step k. In Section 2.1 the relevant governing equations are first derived for the case of patches consisting of only one step (i.e. of uniform thickness). Subsequently these equations are extended to apply to the more general case of multiple stepped patches in section 2.2. For all cases considered we enforce the boundary condition that the adhesive shear strain γ is zero at $x = L/2$.

2.1 Analysis for patch with one step - uniform thickness case

The standard formulation [6,7,8] for this case is given here, as it provides relevant expressions required for the subsequent multistep analysis presented in Section 2.2. In Figure 3 the idealisation of the deformation and force equilibrium for a thin vertical slice of width Δx taken through the patch and plate is given. A one-dimensional idealisation is taken so that displacements in the inner and outer adherends are assumed to be constant across their thicknesses respectively. For the adhesive a uniform shear deformation is assumed across its thickness. For any position x, the tensile forces per unit width of the inner adherend (plate) and outer adherend (patch) are denoted T_i and T_o respectively, and τ is the shear stress in the adhesive layer per unit depth. Also δ_i and δ_o are the displacements in the x direction of the inner and outer adherends respectively. For the thin vertical slice, the equilibrium of forces in the x-direction for the outer and inner material respectively, gives the following two equations

$$\frac{dT_o}{dx} - \tau = 0 \tag{1}$$

$$\frac{dT_i}{dx} + 2\tau = 0 \tag{2}$$

Assuming one dimensional linear elastic stress-strain relations, we can write the strains ε_o, and ε_i for the outer and inner adherends respectively as

$$\varepsilon_o = \frac{d\delta_o}{dx} = \frac{T_o}{E_o t_o} \tag{3}$$

$$\varepsilon_i = \frac{d\delta_i}{dx} = \frac{T_i}{E_i t_i} \tag{4}$$

where E_o and E_i are the elastic moduli of the outer and inner adherends respectively. Since the shear strain γ in the adhesive is assumed to be constant across its thickness η, it is given by

$$\gamma = \frac{\delta_o - \delta_i}{\eta} \tag{5}$$

Differentiating Equation (5) with respect to x and substituting from Equations (3) and (4) gives

$$\frac{d\gamma}{dx} = \frac{1}{\eta}\left(\frac{T_o}{E_o t_o} - \frac{T_i}{E_i t_i}\right) \tag{6}$$

Differentiating again gives

$$\frac{d^2\gamma}{dx^2} = \frac{1}{\eta}\left(\frac{1}{E_o t_o}\frac{dT_o}{dx} - \frac{1}{E_i t_i}\frac{dT_i}{dx}\right) \tag{7}$$

and substituting from Equations (1) and (2) gives

$$\frac{d^2\gamma}{dx^2} = \frac{1}{\eta}\left(\frac{1}{E_o t_o} + \frac{2}{E_i t_i}\right)\tau \tag{8a}$$

Or in terms of shear strain we have

$$\frac{d^2\gamma}{dx^2} = \frac{G}{\eta}\left(\frac{1}{E_o t_o} + \frac{2}{E_i t_i}\right)\gamma \tag{8b}$$

where G is the shear modulus. This can be written as

$$\frac{d^2\gamma}{dx^2} - A^2\gamma = 0 \tag{9}$$

where

$$A = \left[\frac{G}{\eta}\left(\frac{1}{E_o t_o} + \frac{2}{E_i t_i}\right)\right]^{\frac{1}{2}} \tag{10}$$

The general solution to Equation (9) is

$$\gamma = C_1 e^{-Ax} + C_2 e^{+Ax} \tag{11}$$

where C_1 and C_2 can be determined from the boundary conditions.

At the end of the patch, $x = 0$, and we have from equilibrium that

$$T_i(0) = P \tag{12a}$$

and

$$T_o(0) = 0 \tag{12b}$$

Hence substituting Equations (12a) and (12b) into Equation (6) gives

$$\left.\frac{d\gamma}{dx}\right|_{x=0} = \frac{-P}{\eta E_i t_i} \tag{13}$$

Differentiating (11) and equating to Equation (13) gives

$$-AC_1 + AC_2 = \frac{-P}{\eta E_i t_i} \tag{14a}$$

so that

$$C_2 = C_1 - \frac{P}{A\eta E_i t_i} \tag{14b}$$

At the centre of the patch, $x = L/2$, we have the boundary condition

$$\gamma(L/2) = 0 \tag{15a}$$

and from the comparison between Equation (11) and Equation (15a) we have

$$C_1 e^{-AL/2} + C_2 e^{+AL/2} = 0 \tag{15b}$$

giving

$$C_1 = -e^{AL} C_2 \tag{15c}$$

and then substituting Equation (14b) into Equation (15c) gives

$$C_1 = \frac{Pe^{AL/2}}{2\eta E_i t_i A \cosh(AL/2)} \tag{16}$$

Hence from Equations (11), (15c) and (16) the shear strain in the adhesive is given by

$$\gamma(x) = \frac{-P\sinh(Ax - AL/2)}{\eta E_i t_i A \cosh(AL/2)} \tag{17}$$

and using linear stress-strain relations we can also can write the shear stress in the adhesive as

$$\tau(x) = \frac{-GP\sinh(Ax - AL/2)}{\eta E_i t_i A \cosh(AL/2)} \tag{18}$$

2.2 Analysis for patch with multiple steps

2.2.1 Formulation

For the case where the patch has multiple steps, Equations (6) to (11) derived in Section 2.1 are appropriate when interpreted as representing the shear stresses in each step separately. However the boundary conditions at the ends of each step are now different to those presented previously. Referring to the geometry and notation given in Figure 2, we can consider an arbitrary step, k for which we have from Equations (11)

$$\gamma^k = C_1^k e^{-A^k x} + C_2^k e^{+A^k x} \tag{19}$$

Differentiating Equation (19) and equating to Equation (6) we have

$$-A^k C_1^k e^{-A^k x} + A^k C_2^k e^{+A^k x} = \frac{1}{\eta}\left(\frac{T_o^k}{E_o^k t_o^k} - \frac{T_i}{E_i t_i}\right) \tag{20}$$

It is also known from horizontal force equilibrium that at any section x through the patched plate, the following condition applies

$$2T_o^k + T_i = P \tag{21}$$

Substituting from Equation (21) into (20) gives the load in the step, T_o^k , as

$$T_o^k = \left(\frac{1}{E_o^k t_o^k} + \frac{2}{E_i t_i}\right)^{-1}\left(-A^k \eta C_1^k e^{-A^k x} + A^k \eta C_2^k e^{A^k x} + \frac{P}{E_i t_i}\right) \tag{22}$$

Due to equilibrium we require that the force T_o^k is continuous where one step ends and another begins. Also, the shear strain γ has to be continuous at this location from kinetic considerations.

Hence at the beginning of step k, (i.e. at $x = x^{k-1}$), we have

$$\gamma^{k-1} = \gamma^k \tag{23}$$

$$T_o^{k-1} = T_o^k \tag{24}$$

while at the at the end of step k, (i.e. at $x = x^k$), we have

$$\gamma^k = \gamma^{k+1} \tag{25}$$

$$T_o^k = T_o^{k+1} \tag{26}$$

Two further boundary conditions are also known, namely

$$T_o(0)=0 \tag{27}$$

$$\gamma(L/2)=0 \tag{28}$$

Using the continuity and boundary conditions given by Equations (23)-(28) in conjunction with Equations (19) and (22), yields a set of simultaneous linear equations which can be solved for the coefficients C_1^k and C_2^k for $k = 1, n$. The adhesive shear strain distribution can be readily evaluated once the coefficients are determined from Equation (19).

2.2.2 Numerical Solution

A computer program was written in the C++ language to solve for the adhesive shear strains for prescribed geometric and material input data (Appendix A). To avoid the need for matrix manipulation in the code, an simple iterative numerical solution procedure was implemented to solve for the shear strain distribution. In this approach we start in the first step, and use the condition $T_o(0)=0$, and take an arbitrary assumed

value of $\gamma(0)$. From these two initial values, C_1 and C_2 are estimated for the first step using Equations (19) and (22) and consequently the values of T_o and $\gamma(x)$ at the other end of the first step are determined from these two equations. From Equations (23), (24), (19) and (22) values for T_o and γ are used to determine C_1 and C_2 in the next step, and the whole process is repeated for every subsequent step until an estimated value for $\gamma(L/2)$ is found at the center of the patch. If the required condition $\gamma(L/2)=0$ at the center of the patch is not satisfied, then another value of $\gamma(0)$ is assumed and the process is repeated to determine a new estimate of $\gamma(L/2)$. By using an interval halving technique to determine improved estimates for $\gamma(0)$, the method is repeated until the correct value of $\gamma(L/2) = 0$ is obtained. A suitable upper bound first estimate for $\gamma(0)$ can be obtained assuming the patch has uniform thickness, and typically a converged solution is obtained after approximately 60 iterations with a typical accuracy in shear strain of 10^{-6}.

2.3 Estimate for required first step length

It is helpful here to obtain an estimate of a suitable first step length, x_1, which can be used in the numerical solution method. We wish to determine approximately the distance from the beginning of the first step such that the peak shear strain has decayed to almost zero (assuming there are no other steps). Hence if the next step were started here, its presence would have minimal effect on the magnitude of the peak at the end of the patch. To determine the required length we make use of the single step formulation as given in Section 2.1. At the beginning of the first step, the shear strain from Equation (17) is given by

$$\gamma(0) = \frac{P\sinh(A^k L/2)}{\eta E_i t_i A^k \cosh(A^k L/2)} \tag{29}$$

At the position $x = x_1$, we have from Equation (17)

$$\gamma(x_1) = \frac{-P\sinh(A^k x_1 - A^k L/2)}{\eta E_i t_i A^k \cosh(A^k L/2)} \tag{30}$$

Choosing the case where the strain value $\gamma(x_1)$ has reduced to 1% of the peak value, $\gamma(0)$, we have from Equations (29) and (30)

$$0.01 = \frac{-\sinh(A^k x_1 - A^k L/2)}{\sinh(A^k L/2)} \tag{31}$$

which gives

$$0.01 = \frac{-(e^{(A^k x_1 - A^k L/2)} - e^{-(A^k x_1 - A^k L/2)})}{e^{A^k L/2} - e^{-A^k L/2}} \tag{32}$$

Rearranging Equation (32) and setting $e^{-A^k L/2} = 0$, we have since $x_1 << L/2$, that

$$0.01 = e^{-A^k x_1} \tag{33}$$

which gives the required length of the first step as

$$x_1 = \frac{5}{A^k} \tag{34}$$

2.4 Lower bound for peak shear strain due to patch length

It is convenient here to consider one theoretical lower bound on adhesive shear strain. This bound equates to the case where the shear strain is uniform along the patch (except for a region close to $x = L/2$, where the strain must vanish to zero). In this case for a patch of sufficient length, the load transferred to the patch at the center of its length is

$$T_o = P\left(\frac{E_o t_o}{E_i t_i + 2E_o t_o}\right) \tag{35}$$

This is also the load that the adhesive must transmit by shear deformation over the half length of the patch. Hence for a constant adhesive shear stress over this length we have

$$\tau = \frac{T_o}{L/2} \tag{36}$$

Using Equation (8), and substituting Equation (35) into Equation (36) we have one lower bound for a uniform shear strain distribution as

$$\gamma = \frac{2P}{GL}\left(\frac{E_o t_o}{E_i t_i + 2E_o t_o}\right) \tag{37}$$

2.5 Lower bound for peak shear strain due to stiffness of first step

There is also another lower bound for the value of the first peak of the shear strain in the adhesive. This is the peak value of shear strain corresponding to the case where there is a long first step, which can be estimated by putting $x = 0$ in Equation (17) and letting L tend to infinity. This gives the estimated lower bound as

$$\gamma = \frac{-P}{\eta E_i t_i A^k} \tag{38}$$

noting that for this first step $k = 1$.

3. Numerical analyses and results

In this Section the formulation presented in Section 2 is applied to a number of illustrative problems. These examples were chosen to demonstrate the scope for reducing the peak adhesive shear strains occurring at the end of a typical multilayer composite patch bonded to a plate. As previously explained, a typical stepped patch can consist of multiple laminae as shown schematically in Figure 4. It is important to make clear here that each step consists of one or multiple laminae. Hence the value of the effective stiffness, $E_o^k t_o^k$, as used in the formulation of Section 2.2 will be different for each step, and will depend on the properties of the individual lamina within the step. For a particular step, the effective stiffness is given by

$$E_o^k t_o^k = \sum_{l=1}^{l=m} E_l t_l \tag{39}$$

where the subscript l refers to an individual lamina, and m denotes the number of laminae in the step. To provide a meaningful comparison for the various patch configurations presented, a number of parameters and boundary conditions were kept the same for all cases, namely: (i) remote loading (ii) plate properties, (iii) the adhesive shear stress has reduced to zero at the centre of the patch, $x = L/2$, and (iv) the maximum effective stiffness $E_o t_o$ for the patch is equivalent to that for ten unidirectional boron/epoxy laminae. Hence the same amount of load was transferred to the patch for each analysis case.

3.1 Input data

The remote loading was $P =$ 2000 kN/m, and the material and geometric properties were as follows:

(i) Aluminium plate

Young's modulus	$E_i = 71\,000$ MPa
thickness	$t_i =$ 6 mm

(ii) Boron/epoxy lamina used to compose multi-layer patch

Young's modulus (unidirectional orientation)	E_o = 208 000 MPa
Young's modulus (cross-ply orientation)	E_o = 20 800 MPa
thickness	t = 0.13 mm
maximum length	$L =$ 80 mm

(iii) Typical structural adhesive

shear modulus	$G =$ 590 MPa
thickness	$\eta =$ 0.1 mm

For comparison purposes these properties were chosen to correspond to those used by Chalkley [3].

3.2 Preliminary analysis and benchmark cases

In Figure 5 the adhesive shear strain distribution is shown for the case where each patch consists of one step only, comprising of ten unidirectional laminae. Hence the patch has uniform thickness of 1.3 mm. The solution for this case provides an upper bound to the value of the peak adhesive shear strain for stepped patches. The peak and exponential decay, in the adhesive shear strain distribution at the end of the patch can be readily seen. The adhesive shear strain distribution for the case where there are two steps of equal length is shown in Figure 6. Here the first step of the patch is 0.65 mm thick (consisting of five laminae) and thereafter the total patch thickness is 1.3 mm (consisting of ten laminae). It can be seen that this configuration provides a significant improvement as compared to the case represented in Figure 5.

The adhesive shear strain distribution for the case of 10 equal step heights of 0.13 mm, and a uniform step length of 3 mm is given in Figure 7. Here each step height is equivalent to one lamina thickness. Results for this patch case have been given by Chalkley [3], and the present results are in very close agreement with his finite element and analytical results. This stepping scheme is typical of that recommended by the RAAF Engineering Standard on bonded repairs. There are a number of points of interest arising from this case. Firstly, while the peak shear strain is significantly reduced as compared to the single step case, the highly localised peak in adhesive shear strain at the end of the patch is also clearly evident. Secondly there is significant interaction between the first and second peaks. Thirdly, the maximum shear strain in the adhesive is significantly greater than the lower bound of $\gamma = 0.0237$ determined from Equation (32) for the given length of the patch. Hence it is evident that there is scope for minimising the magnitude of the peak strain.

3.3 Unidirectional patch with long thin first step

One obvious approach for minimising the interaction of the first and second peak, for a patch comprising unidirectional laminae is to use a longer first step. Figure 8 shows the adhesive shear strain distribution when the patch has two steps, the second arbitrarily chosen in this example at 15 mm from the first step. The first step is very thin, of thickness 0.13 mm (typical of one lamina), while the total patch thickness after the step at 15 mm is 1.3 mm. Although the first step is very thin compared to the second step, the strain pattern becomes quite different from not having this first thin step. It can be seen that there is minimal interaction between the two peaks (which

occur at the beginning) at each step, hence the first peak has been reduced to its limiting value for the given load and appropriate material properties. With regard to the magnitude of the second peak, it should be emphasised that the total area under the curve is proportional to the load transferred to the patch material. Since the first step has taken up some of the load to be transferred, the area under the curve from the second step to the center of the patch will be less than the total area under the curve in Figure 5. Therefore, as expected the peak at the second step is significantly less than the peak shown in Figure 5. Clearly, splitting the second step into multiple steps would reduce the value of the second peak.

We now consider the case where we again have a long first step but now the second step for the case discussed above is further split into 9 steps. The adhesive shear strain distribution for such a case is given in Figure 9. Here the patch has a thin long first step of length 19 mm consisting of 1 lamina. Subsequent steps each consist of one lamina and have equal lengths of 0.5 mm, except for the last step which has a length of 17 mm to take up the remaining half length of the patch. It is evident that this method of having maximal length for the first thin step gives a reduction of about 20% in the peak shear strain at the edge of the patch, as compared to the standard method given represented in Figure 5. It is informative to repeat the case as given in Figure 9, except that we reduce the first step length from 19 mm to 14 mm, and adjusting the last step to keep half length of the patch unchanged. The corresponding results are given in Figure 10. Here it can be seen that there has been little change in maximum shear strain at the end of the patch due to the reduced first step length. This demonstrates that once the first step length has exceeded some minimum value (i.e. transfer length) there is little additional benefit in increasing it.

3.4 Patch incorporating some cross-ply laminae

The previous examples show that once its first step length exceeds a certain transfer length, a stepped bonded patch with uniform lamina thickness has its the peak value of adhesive shear strain occurring at its beginning, and in that first step. In terms of patch properties, this peak depends strongly on $E_o t_o$ of the first step. Hence to reduce the magnitude of the peak further the value of the effective stiffness $E_o t_o$ of the first step must be lowered. One possible method of achieving this is to replace the unidirectional lamina in the first step with a cross-ply, so that the new $E_o t_o$ value is equivalent to about one tenth of that for a typical unidirectional lamina. With this in mind one possible general stepping arrangement can be proposed as follows: (i) first step consists of one cross-ply lamina, (ii) a combination of cross-ply and unidirectional lamina are used for the remaining steps, and (iii) non-uniform step lengths are used. To allow a direct comparison with the previous cases, the maximum value of $E_o t_o$ for the patch is kept the same as previously. One way of meeting this requirement is easy to replace the first unidirectional step, used in the previous cases, with ten cross-ply laminae further split into three steps, while keeping the step heights of the remaining 9 steps unchanged (each consisting of one lamina). Hence this patch consists of 11 steps, with a combination of 19 unidirectional and cross-ply lamina with the geometric details as given in Table 1. Here the step lengths were chosen such that the highest shear strain was minimised. The resulting shear strain distribution is shown in Figure

11, and it can be seen that the peak strain value has been reduced by about 60 % from that given by Figure 5. Due to the dominant effect of the stiffness of the first step, it is believed that a similar reduction can be achieved by adding just one extra layer of cross ply lamina, underneath a typical AMRL unidirectional patch, which would only require 11 lamina in total.

Table 1. Geometric details for patch on one side of plate for non-uniform stepping case.

Step number	Step length (mm)	Layup order of laminae within step (C refers to cross-ply laminae) (U refers to unidirectional laminae)
1	1.0	1C
2	2.0	3C
3	5.0	6C
4	4.5	10C,1U
5	3.5	10C,2U
6	2.5	10C,3U
7	1.5	10C,4U
8	1.5	10C,5U
9	1.5	10C,6U
10	0.5	10C,7U
11	16.5	10C,9U

3.5 Comments on achieving lower bound for peak shear strain

For the case shown in Figure 11 (where cross-ply laminae are used), the value of the maximum shear strain has been lowered to be approximately twice the theoretically lowest bound as given by Equation (37), (where the total patch length has been constrained to 80 mm). If this lower bound for the maximum shear strain is to be approached, the following two conditions must be satisfied: (i) $E_o t_o$ of the first ply step must be reduced by about another factor of 4 below that already achieved with the use of cross-ply material (see Equation (17); this requirement is needed irrespective of the first step length), (ii) a larger number of subsequent steps (and laminae) are required for the patch to give more gradual changes in $E_o t_o$ across step interfaces. It is important to note that when the total patch length is increased, condition (ii) can be relaxed, but not condition (i).

4. Discussion and recommendations

4.1 Lower bounds for peak shear strain

In general there are two possible lower bounds for the peak value of the shear strain. The first lower bound is given by Equation (32) and is determined by the total length and relative stiffness of the patch. The second lower bound is given by Equation (38) and is determined by the value $E_o t_o$ of the first step. The second lower bound is almost

independent of the length of the first step (once a certain value is exceeded) and of the values $E_o t_o$'s of subsequent steps of the patch. An accurate value of the peak shear strain at the edge of the patch that takes into account the interaction from the second step can be obtained using the formulation given in Section 2 of this report, and this value will always be higher than the above two lower bounds mentioned above. The ideal design would have the two lower bounds equal and have the peak shear strain approach the lower bound value. Three possible patch design scenarios relating to the patch stiffness properties are given in the following sections.

4.2 Patch configuration options

4.2.1 Patch consisting of unidirectional laminae only

For this case all lamina thicknesses are the same and the best stepping procedure is to have a relatively long first step (Equation (34)) while successive steps can be quite close together. The lengths should be chosen to minimise the strain peaks by following the method given in this report. Typical reductions in peak strain are about 20%, as compared to that obtained by following the RAAF Engineering Standard [2] which employs uniform stepping. This 20% reduction is for the typical case of an aluminium plate patched on each side with one patch made from boron/epoxy laminae of 0.13 mm thickness each.

4.2.2 Patch consisting of cross-ply and unidirectional laminae of the same material

Here at least the first step of the patch should consist of one or more cross-ply laminae with the aim of lowering the second lower bound determined from Equation (38), so that it is nearly equal to the first lower bound determined by Equation (32). Subsequent steps can consist of a combination of unidirectional laminae and or cross-plys. If required, the first few subsequent steps can consist of solely cross-plys. Also, the first step length should be longer than the non-interaction length as estimated by Equation (34). Due to the use of cross-ply material for these steps, the non-interaction length is significantly less than that corresponding to the use of unidirectional material. For typical repair cases, this method can reduce the peak shear strain by about 60% from the value otherwise obtained with the uniform stepping of 3 mm. If the number of laminae is to be kept to a minimum, more interaction between steps can be allowed and the reduction in shear strain can still be maintained at close to the value of 60%.

4.2.3 Patch consisting of cross-ply or unidirectional laminae of different materials

This case is not analysed in this report, however it clearly offers the potential of a gradual change in $E_o t_o$ if desired, while minimising the total length over which the stepping needs to occur. The minimum attainable value of peak shear strains will depend on $E_o t_o$ of the first step and on the total permissible length of the patch. Depending on the chosen material properties, this approach has the potential of

enabling a stress distribution approaching the lower bound as given by Equation (38) to be achieved. It should also be noted that there may be practical difficulties in the manufacture/application of a patch with cross-ply laminae of the different materials, and this needs to be taken into account.

4.3 Implications for reducing stress concentration in plate

It has become apparent in recent years that bonded patches generally create a stress concentration in the repaired material immediately outside the patched region. This effect can be of particular concern in some situations since it can cause unexpected failure immediately outside the patched region [9]. It should be noted that this effect has not been taken into account by the RAAF Engineering Standard C5033, although it is planned for inclusion. It is clear from the results presented in this report, that minimising the adhesive strains is essentially equivalent to smoothing the load transfer from the plate to the patch. This is also logically desirable from the viewpoint of structural integrity of the repaired structure. It is reasonable to expect that minimising the adhesive shear strains will naturally lead to a reduction in the stress concentration outside the patch. Inspection of the results given in reference [5] gives evidence supporting this conjecture, as does recent analytical work [10] and two dimensional finite element stress analysis [11].

5. Conclusions

An elastic one-dimensional model has been used to examine the effect of various parameters in the optimal lamina stepping for fibre composite laminated patches used in the repair of a plate. The model has been applied to a typical case of an aluminium plate repaired using a boron/epoxy patch on each of its sides. It is shown that to gain an improvement over the standard uniform stepping method as recommended in the RAAF Engineering Standard C5033 (for a typical boron/epoxy laminated unidirectional patch), the first step adjacent to the edge needs to have a much longer step length than the remaining steps. If this method is followed there should be a reduction in peak shear strain of the order of 20%. For the case where cross-ply laminae can be used (typically in the first two steps) in conjunction with unidirectional laminae, the maximum shear strain in the adhesive layer can be reduced by about 60%. It is also demonstrated that the numerical method presented in this work can be used for the design of an optimised stepped patch of a given length.

Reducing peak adhesive shear strains leads to a smoother transition of load from the plate to the patches. This implies a patch design that reduces peak adhesive strain will also reduce the undesirable stress concentration in the plate immediately outside the patched region, (which often occurs with typical application of standard uniform stepped patches). Recent analytical and two dimensional finite element stress analysis has confirmed this hypothesis and also highlight the importance of correct patch stepping.

6. Acknowledgments

The authors wish to acknowledge the encouragement and advice given by Dr F. Rose, and the helpful comments of Dr A. A. Baker, Dr. R. J. Chester, P. Chalkley and Mr M. Davis. The authors also wish to thank P. Chalkley for access to the computer program developed in Reference [3], which was used in the present work for comparison purposes, for the uniform stepping case.

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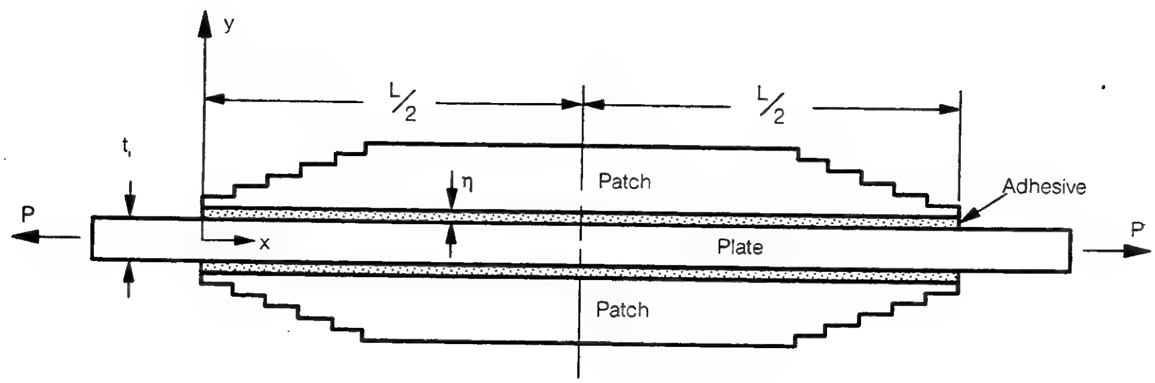


Figure 1. General arrangement for plate with bonded symmetric stepped patches.

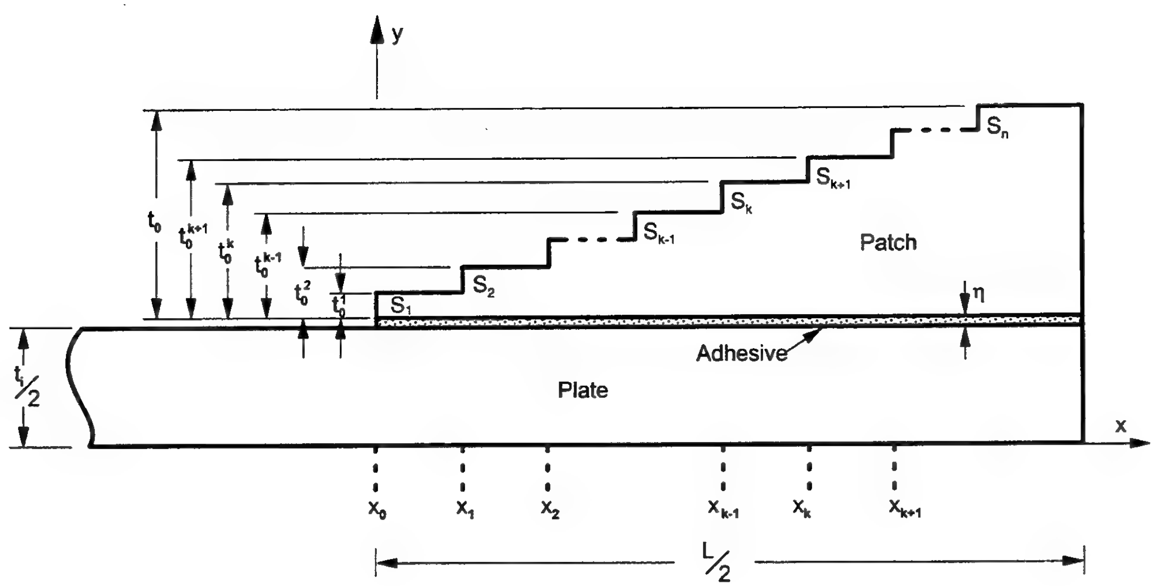
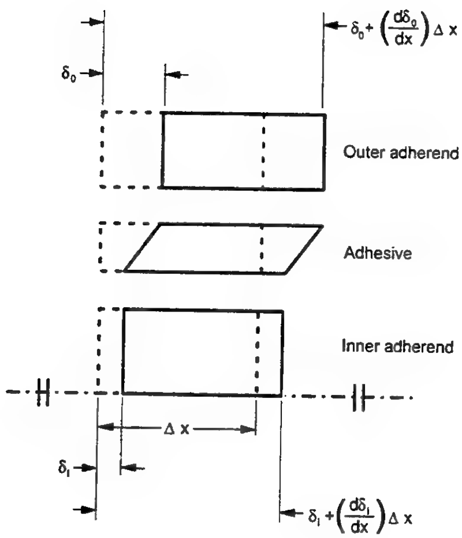
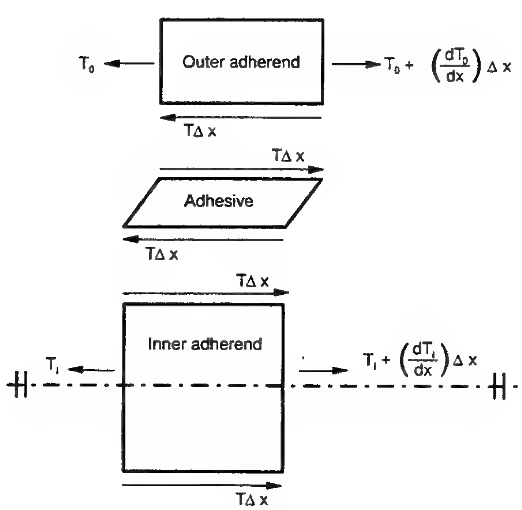


Figure 2. Notation and geometric definitions for stepped region as used in analytical model.



(a) displaced position (solid line) and reference position (dashed line)



(b) force equilibrium

Figure 3. Schematic for model formulation based on horizontal displacements and force equilibrium for vertical slice through plate with bonded patches (only top half of symmetric configuration shown).

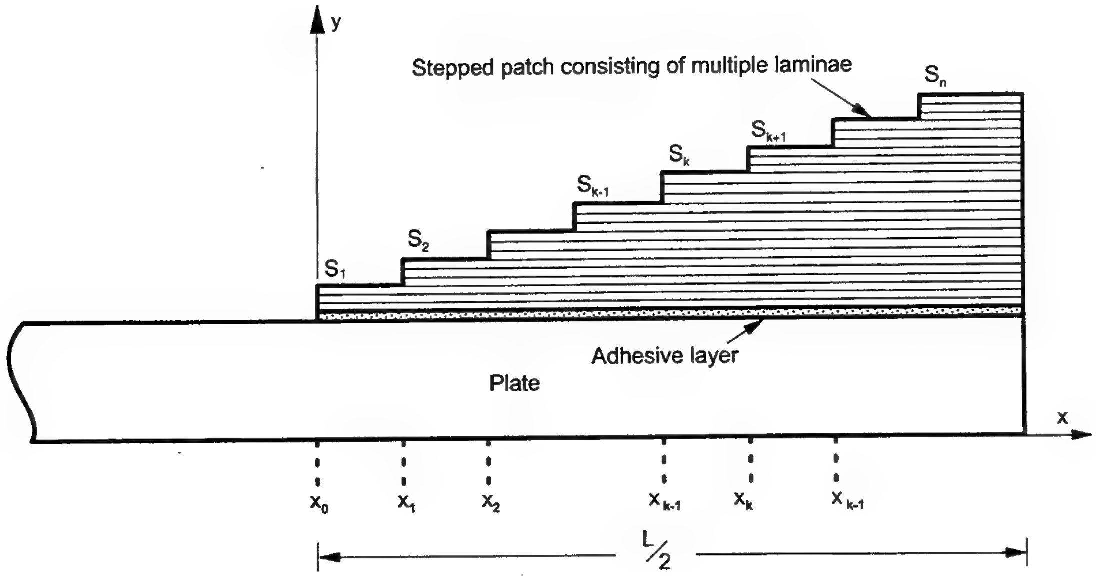


Figure 4. Schematic showing steps consisting of multiple laminae.

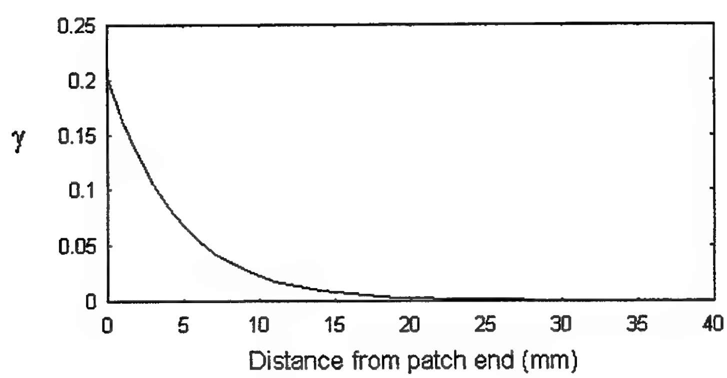


Figure 5. Adhesive shear strain (γ) distribution for case of patch with one step.

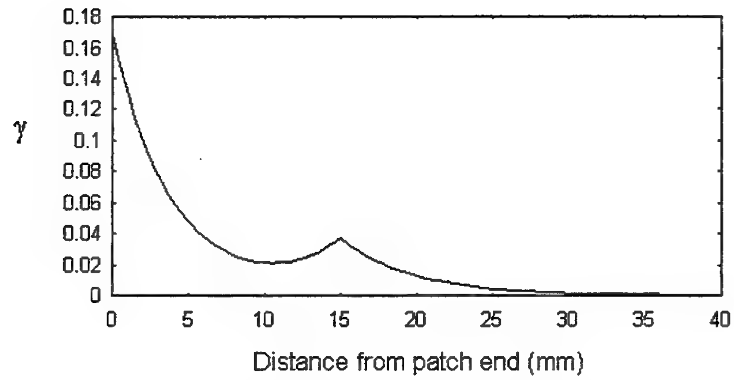


Figure 6. *Adhesive shear strain (γ) distribution where the patch consists of two steps, each of the same height.*

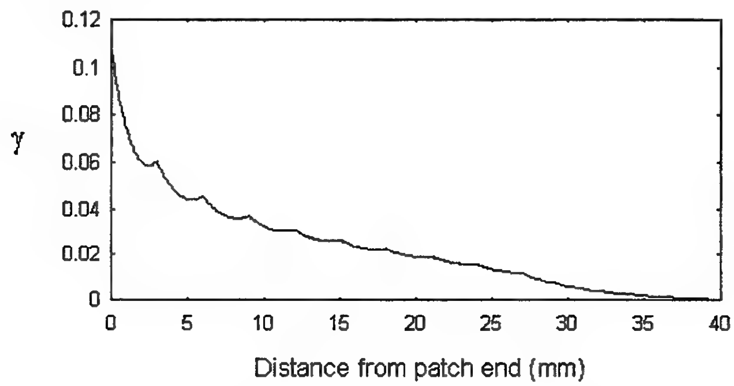


Figure 7. *Adhesive shear strain (γ) distribution where the patch consists of ten steps of equal height with a uniform step length of 3 mm.*

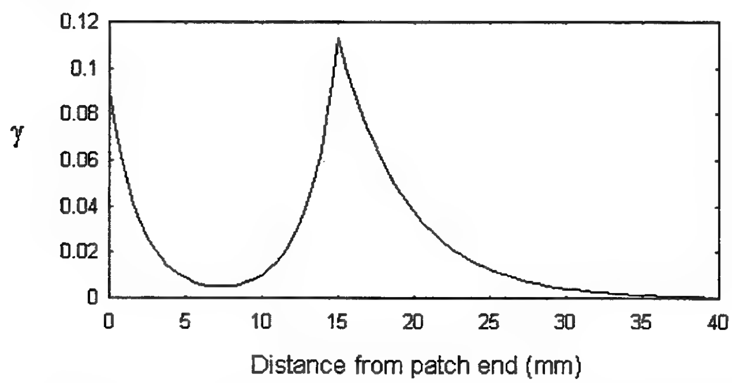


Figure 8. *Adhesive shear strain (γ) distribution for the case where the patch consists of two steps, the first being one lamina thick.*

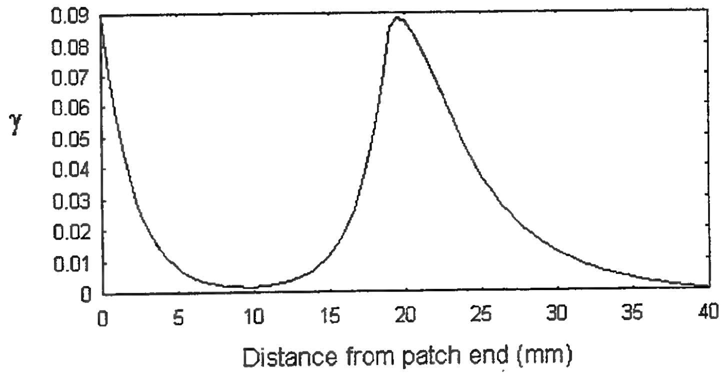


Figure 9. *Adhesive shear strain (γ) distribution where the patch consists of ten equal step heights and non-uniform step lengths, with first step length of 19 mm.*

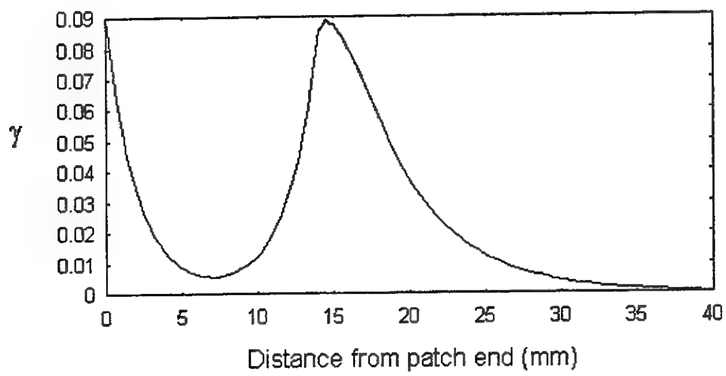


Figure 10. *Adhesive shear strain (γ) distribution where the patch consists of ten equal step heights and non-uniform step lengths, with first step length of 14 mm.*

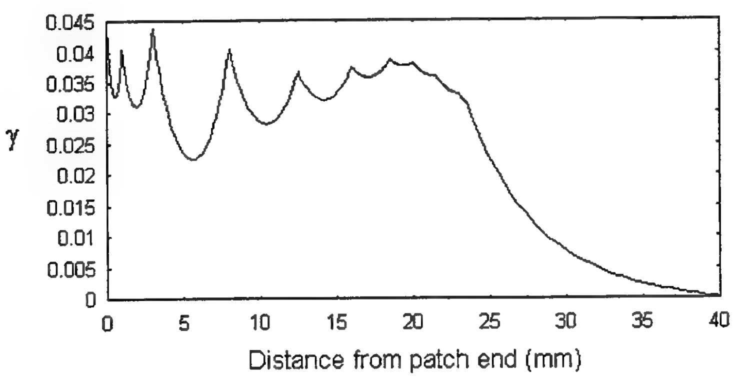


Figure 11. *Adhesive shear strain (γ) distribution for the case of 11 unequal step thicknesses with non-uniform step lengths, and a combination of 10 cross-ply and 9 unidirectional laminae.*

Appendix A

This appendix contains a listing of a computer program (written in the C++ language), which can be used to determine the adhesive shear strain distribution for a plate with symmetric bonded stepped patches. The numerical solution algorithm used is as discussed in Section 2.2.2.

```
//PATCH.CPP, author: Ton Tran-Cong,
#include <stdio.h>
#include <math.h>
#include <b:\tranprog\cpp\solvegam.cpp>

void main()
{
   FILE *stream;
   FILE *stream1;
   stream  = fopen("C:\\USER\\TRANPROG\\CPP\\PATCH.OUT", "w+");
   /* open the output file*/
   stream1 = fopen("C:\\USER\\TRANPROG\\CPP\\PATCH.DA1", "w+");
   /* open a file for GNUPLOT */
   printf("\n\n Program PATCH.CPP to compute the adhesive shear stresses \n");
   fprintf(stream,"\n\n Program PATCH.CPP to compute the adhesive shear stresses \n");
   printf("          for task 212227c    \n");
   fprintf(stream,"              for task 212227c    \n");
   fprintf(stream1," ");
   /* without this line, stream1 has no output, it is strange! */
   fclose(stream);
   fclose(stream1);/* close the file */

   double E_i,E_o,G;
   double eta,t_i,t_o,P,L,x0;
   double T_i,T_o;

   E_i=71000;
   E_o=208000;
   G=590;
   eta=0.1;
   t_i=6.36;
   P=2000;
   solvegam(E_i,E_o,G,eta,t_i,P);
}
//function SOLVEGAM.CPP, to solve for gamma value at x=0;
//author Ton Tran-Cong, 13Jul95;
//such that T_o vanishes at x=infinity;
#include <stdio.h>
#include <math.h>
#include <b:\tranprog\cpp\tension.cpp>

double solvegam(double E_i,double E_o,double G,double eta,double t_i,double P)
{
   int it,is,istep=3,iter=2,iwrt=0;
   double t_o,L,x0=0;
   double tstep[12],lstep[12];
//unlike Pascal, C++ has array starting from tstep[0],
//Borland C++, Programmer's Guide p.39.
   long int nstep=40;
   double T_i,T_o,gamma=0;
   double gammal=0.,gammah=0.,gammai=0.;
   double A,A2;
   iter=60;
   istep=10;
```

```
        lstep[1]=3;
        tstep[1]=0.130;
        lstep[2]=3;
        tstep[2]=0.260;
        lstep[3]=3;
        tstep[3]=0.390;
        lstep[4]=3;
        tstep[4]=0.520;
        lstep[5]=3;
        tstep[5]=0.650;
        lstep[6]=3;
        tstep[6]=0.78;
        lstep[7]=3;
        tstep[7]=0.91;
        lstep[8]=3;
        tstep[8]=1.04;
        lstep[9]=3;
        tstep[9]=1.17;
        lstep[10]=13;
        tstep[10]=1.30;
        lstep[11]=10.;
        tstep[11]=1.352;

        T_o=0.;
        T_i=P;
        L=lstep[1];
        t_o=tstep[1];
        A2=(G/eta)*(1./(E_o*t_o)+2./(E_i*t_i));
        A=sqrt(A2);
        gamma=P/(eta*A*E_i*t_i);/* standard value for infinite patches */
        gammal=gamma;
        gammah=2.*gamma;

        for (it=1;it<=iter;it++)
        {
        iwrt=0;
        if (it==iter) iwrt=1;
        gammai=0.5*(gammal+gammah);
        edgedata.gamma=gammai;
        edgedata.T_o=0.;
        x0=0.;

        for (is=1;is<=istep;is++)
        {
        gamma=edgedata.gamma;
        T_o=edgedata.T_o;
        T_i=P-2.*T_o;
        L=lstep[is];
        t_o=tstep[is];
        A2=(G/eta)*(1/(E_o*t_o)+2/(E_i*t_i));
        A=sqrt(A2);
        if(is>1) x0=x0+lstep[is-1];
        tension(G,E_i,E_o,eta,t_i,t_o,T_i,T_o,x0,gamma,L, nstep, iwrt);
        };

        gamma=edgedata.gamma;
        if (gamma>0) {gammah=gammai;};
        if (gamma<0) {gammal=gammai;};
        };
        return gammai;
    }
//function TENSION.CPP to calculate the tension at the other end of a
//patch from the given data of one end.
```

```
#include <stdio.h>
#include <math.h>

struct edgestr {double G; double E_i; double E_o;\
        double eta; double t_i; double t_o; double T_i; double T_o; double x0;\
        double gamma;double L; long int nstep;int iwrt;} edgedata;

edgestr tension(double G, double E_i, double E_o,\
        double eta, double t_i, double t_o, double T_i, double T_o, double x0,\
        double gamma,double L, long int nstep, int iwrt)
        /* tension return a pointer to structure edgestr, PG p.67 */

{
        FILE *stream;
        FILE *stream1;

        if (iwrt==1) stream  = fopen("C:\\USER\\TRANPROG\\CPP\\PATCH.OUT", "a+");
        /* open the output file*/
        if (iwrt==1) stream1 = fopen("C:\\USER\\TRANPROG\\CPP\\PATCH.DA1", "a+");
        /* open a file for GNUPLOT */

double f1,f2;
long int i,j;
double gamma0,dgamma0,d2gamma0;
double gamma1,dgamma1,d2gamma1;
double gamma2,dgamma2,d2gamma2;
double x,dx,dgamma,d2gamma,tau,delta_i,delta_o;
double P=1,A,A2;
double C_1,C_2;

        /* t_i,t_o are thicknesses of inner and    outer layers*/
        A2=(G/eta)*(1/(E_o*t_o)+2/(E_i*t_i));
        A=sqrt(A2);
        x=x0;

        C_1=0.5*exp(A*x)*gamma;
        C_1=C_1-exp(A*x)/(2*eta*A)*(T_o/(E_o*t_o)-T_i/(E_i*t_i));

        C_2=0.5*exp(-A*x)*gamma;
        C_2=C_2+exp(-A*x)/(2*eta*A)*(T_o/(E_o*t_o)-T_i/(E_i*t_i));
        P=T_i+2*T_o;

        /* t_i,t_o are thicknesses of inner and
        outer layers at 0<x<(L/2) */
        A2=(G/eta)*(1/(E_o*t_o)+2/(E_i*t_i));
        A=sqrt(A2);
        x=x0;
        d2gamma=A2*gamma;
        dgamma=(A2/G)*T_o-(T_i+2*T_o)/(eta*E_i*t_i);
        printf("\nA=%e \n",A);
        if (iwrt==1) fprintf(stream,"\nA=%e \n",A);/* write data onto the output file */
        if (iwrt==1) fprintf(stream1," ");/* write data onto the output file */
        printf(         "\n      x          gamma          dgamma\
           d2gamma          T_o\n");
        if (iwrt==1) fprintf(stream,"\n      x          gamma          dgamma\
           d2gamma          T_o\n");
        printf(          "% 9.3f     % 9.3e     % 9.3e     % 9.3e     % 9.3e \n",
           x,gamma,dgamma,d2gamma,T_o);
        iwrt=iwrt;
        if (iwrt==1) fprintf(stream, "% 9.3f     % 9.3e     % 9.3e     % 9.3e     % 9.3e \n",
           x,gamma,dgamma,d2gamma,T_o);
        if (iwrt==1) fprintf(stream1,"% 9.3f     % 9.3e     % 9.3e     % 9.3e     % 9.3e \n",
           x,gamma,dgamma,d2gamma,T_o);

        x=x0;
```

```
        dx=L /(nstep);
        for (i=1;i<=nstep;i++)
        {x=x+dx;

// modified part using analytical calculation
        gamma=C_1*exp(-A*x)+C_2*exp(A*x);
        T_o=(1/(1/(E_o*t_o)+2/(E_i*t_i)))*\
        (-A*eta*C_1*exp(-A*x)+A*eta*C_2*exp(A*x)+P/(E_i*t_i))   ;
        T_i=-(1/(1/(E_o*t_o)+2/(E_i*t_i)))*\
        (-2*A*eta*C_1*exp(-A*x)+2*A*eta*C_2*exp(A*x)-P/(E_o*t_o))   ;
        d2gamma=A2*gamma;
        dgamma=(A2/G)*T_o-(T_i+2*T_o)/(eta*E_i*t_i);

        printf(         "% 9.3f    % 9.3e    % 9.3e    % 9.3e    % 9.3e \n",
           x,gamma,dgamma,d2gamma,T_o);
        if (iwrt==1) fprintf(stream, "% 9.3f    % 9.3e    % 9.3e    % 9.3e    % 9.3e \n",
           x,gamma,dgamma,d2gamma,T_o);
        if (iwrt==1) fprintf(stream1,"% 9.3f    % 9.3e    % 9.3e    % 9.3e    % 9.3e \n",
           x,gamma,dgamma,d2gamma,T_o);
        };

        if (iwrt==1) fclose(stream);
        if (iwrt==1) fclose(stream1);

        T_o=(dgamma+P/(eta*E_i*t_i))*(G/A2);
        /* we should have T_o=P*E_o*t_o/(2*E_o*t_o+E_i*t_i) at x=infinity
        from the condition of equal strain for all layers after uniform
        strain has been obtained with the vanishment of shear stress. */

// modified part using analytical calculation
        x=x0+L;
        gamma=C_1*exp(-A*x)+C_2*exp(A*x);
        T_o=(1/(1/(E_o*t_o)+2/(E_i*t_i)))*\
        (-A*eta*C_1*exp(-A*x)+A*eta*C_2*exp(A*x)+P/(E_i*t_i))   ;
        T_i=-(1/(1/(E_o*t_o)+2/(E_i*t_i)))*\
        (-2*A*eta*C_1*exp(-A*x)+2*A*eta*C_2*exp(A*x)-P/(E_o*t_o))   ;

        edgedata.T_o=T_o;
        edgedata.gamma=gamma;
        return edgedata;
}
```

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A mathematical model is presented which defines the adhesive shear strain distribution for an adherend with bonded multilayer reinforcements which are stepped at their ends. In this one-dimensional formulation each step is allowed to be of different thickness and modulus, and of variable step length. A procedure is then given to improve the design of such reinforcements through minimising the peak adhesive shear strain which typically occurs near their stepped ends. It is shown that to achieve a 20% reduction in peak adhesive shear strain for a typical stepped patch consisting of unidirectional laminae, the first step adjacent to the patch end needs to be much longer than the remaining steps. For the case where cross-ply laminae are used in conjunction with unidirectional laminae, the maximum shear strain in the adhesive layer can be reduced by about 60%. The results also indicate that reduced peak adhesive shear strains lead to a smoother transition of load from the plate to the patch. This suggests that a patch design which minimises peak adhesive shear strains will also reduce the undesirable stress concentration in the repaired structure, outside the patched region.

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